CHARACTERISTICS OF THERMAL RADIATION IN AXISYMMETRIC CAVITIES

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Radiation heat transfer in axisymmetric cavities with a lateral surface formed by a truncated cone and plane ends is investigated. A numerical example is considered for a range of cavities. The calculated results are universal and can be used for rapid estimates of radiation heat transfer.

The solution of many engineering problems requires calculation of the radiation heat transfer in cavities of various shapes. We shall consider axisymmetric cavities with a lateral surface formed by a truncated cone; the ends are planes. The investigated cavity and the coordinate system used in the solution are shown schematically in Fig. 1. The emissivities ε_i of each surface bounding the cavity are constant but differ in general; the temperature is an arbitrary function of the coordinate (all parameters are taken to be constant with respect to the angular coordinate φ).

In deriving the system of integral equations describing the process of radiation heat transfer in the cavity we assume that the processes of emission and reflection of radiation energy are diffuse. Using the approach of [1] we set up the radiation energy balance equation for arbitrary elementary areas situated on the inside surface of the cavity. The energy leaving the area with the coordinate r_0 (surface 1, Fig. 1) equals the sum of the self-emission and the reflected emission

$$B(r_0) = \varepsilon_1 \sigma T^4(r_0) + (1-\varepsilon_1) H(r_0). \tag{1}$$

Analogous relationships are set up for the areas belonging to surfaces 2 and 3.

There is a second expression that is paired with (1); it comes from a consideration of the sources of the incident radiation $H(r_0)$:

$$H(r_0) = \int\limits_{x=0}^{L} B(x)\, dF^*_{r_0-x} + \int\limits_{r^*=0}^{R^*} B(r^*)\, dF^*_{r_0-r^*}. \tag{2}$$

In (2) $dF^*_{r_0-x}$ and $dF^*_{r_0-r*}$ are elementary angular coefficients of the area with the coordinate r_0 with respect to the annular elements with the coordinate x (surface 2) and $r*$ (surface 3). We shall derive the formulas determining the $dF*$ later.

Substituting (2) into (1) we obtain

$$B(r_0) = \varepsilon_1 \sigma T^4(r_0) + (1-\varepsilon_1)\left[\int\limits_{x=0}^{L} B(x)\, dF^*_{r_0-x} + \int\limits_{r^*=0}^{R^*} B(r^*)\, dF^*_{r_0-r^*}\right]. \tag{3}$$

The analogous equations for the areas with the coordinates x_0 (surface 2) and r_0^* (surface 3) have the form

$$B(x_0) = \varepsilon_2 \sigma T^4(x_0) + (1-\varepsilon_2)\left[\int\limits_{r=0}^{R} B(r)\, dF^*_{x_0-r} + \int\limits_{x=0}^{L} B(x)\, dF^*_{x_0-x} + \int\limits_{r^*=0}^{R^*} B(r^*)\, dF^*_{x_0-r^*}\right], \tag{4}$$

$$B(r_0^*) = \varepsilon_3 \sigma T^4(r_0^*) + (1-\varepsilon_3)\left[\int\limits_{r=0}^{R} B(r)\, dF^*_{r_0^*-r} + \int\limits_{x=0}^{L} B(x)\, dF^*_{r_0^*-x}\right]. \tag{5}$$

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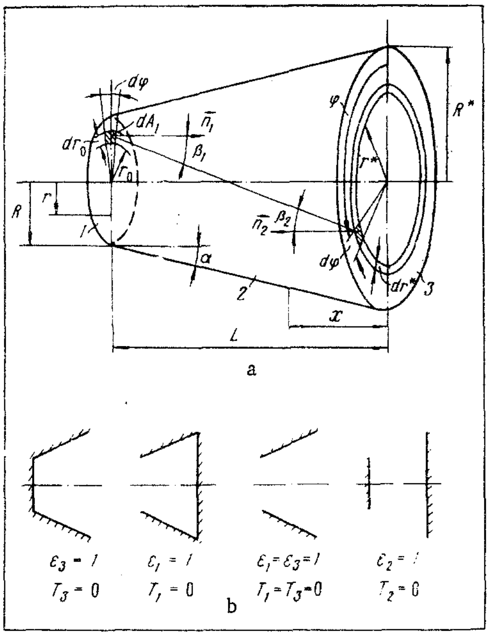

Fig. 1. Schematic representation of the cavities investigated (b) and the coordinate system used (a).

Equations (3)-(5) represent the desired system of integral equations in the unknown functions B.

We take $dF^*_{r_0-r^*}$ as the example for derivation of the formulas for the angular coefficients. Figure 1 shows an annular element and areas on surfaces 3 and 1 respectively. We assume that the temperature $T(r^*)$ is constant within the limits of an annular element. By definition the angular coefficient of the area $dA_1 = r_0 d\varphi dr_0$ with respect to the area $dA_2 = r^* d\varphi dr^*$ is determined by the formula [2]

$$dF_{dA_1-dA_2} = \frac{\cos\beta_1 \cos\beta_2}{\pi s^2}\, dA_2. \tag{6}$$

Summing all areas lying within the annular element, we obtain

$$dF^*_{dA_1-dA_2} = 2\int_0^{\pi} \frac{\cos\beta_1 \cos\beta_2}{\pi s^2}\, dA_2. \tag{7}$$

Evaluating the integral on the right side of (7) we finally get the formula

$$dF^*_{r_0-r^*} = \frac{2L^2 r^* (L^2 + r_0^2 + r^{*2})\, dr^*}{[(L^2 + r_0^2 + r^{*2})^2 - 4r^{*2} r_0^2]^{3/2}}. \tag{8}$$

In like manner we derive formulas for the other angular coefficient occurring in the system (3)-(5); they have the form

$$dF^*_{r_0^*-r} = \frac{2L^2 r (L^2 + r^2 + r_0^{*2})\, dr}{[(L^2 + r^2 + r_0^{*2})^2 - 4r_0^{*2} r^2]^{3/2}}, \tag{9}$$

$$dF^*_{r_0-x} = \frac{2R_1 (L-x)\{[(L-x)^2 + R_1^2 + r_0^2]\, R - 2r_0^2 R_1\}\, dx}{\{[(L-x)^2 + R_1^2 + r_0^2]^2 - 4r_0^2 R_1^2\}^{3/2}}, \tag{10}$$

$$dF^*_{x_0-r} = \frac{2r (L-x_0)\cos\alpha \{[(L-x_0)^2 + R_2^2 + r^2]\, R - 2r^2 R_2\}\, dr}{\{[(L-x_0)^2 + R_2^2 + r^2]^2 - 4r^2 R_2^2\}^{3/2}}, \tag{11}$$

$$dF^*_{r_0^*-x} = \frac{2R_1 x\, [R^* (x^2 + R_1^2 + r_0^{*2}) - 2R_1 r_0^{*2}]\, dx}{[(x^2 + R_1^2 + r_0^{*2})^2 - 4r_0^{*2} R_1^2]^{3/2}}, \tag{12}$$

$$dF^*_{x_0-r^*} = \frac{2x_0 r^* \cos\alpha\, [R^* (x_0^2 + R_2^2 + r^{*2}) - 2R_2 r^{*2}]\, dr^*}{[(x_0^2 + R_2^2 + r^{*2})^2 - 4r^{*2} R_2^2]^{3/2}}, \tag{13}$$

$$dF^*_{x_0-x} = \frac{\cos\alpha}{2R_2}\left\{1 - \frac{|x_0 - x|}{\cos\alpha}\, \frac{\left[\dfrac{(x_0-x)^2}{\cos^2\alpha} + 6R_1R_2\right]}{\left[\dfrac{(x_0-x)^2}{\cos^2\alpha} + 4R_1R_2\right]^{3/2}}\right\} dx. \tag{14}$$

In (10)-(14)

$$R_1 = R + (L-x)\operatorname{tg}\alpha; \quad R_2 = R + (L-x_0)\operatorname{tg}\alpha, \tag{15}$$

the remaining notation will be clear from Fig. 1a.

System (3)-(5) is quite general and may be used to calculate the radiation heat transfer in axisymmetric cavities having shapes other than that shown in Fig. 1a by letting $\varepsilon_i = 1$, $T_i = 0$ ($i = 1, 2, 3$ is the number of the surface). Calculations can be carried out for cavities having the shapes indicated in Fig. 1b. To make the solutions of (3)-(5) universal we refer all linear dimensions occurring in the above formulas to the radius R of the end surface 1 (we shall not use the new designations in the ensuing discussion) and introduce certain functions

$$\varepsilon_a = \frac{B}{\sigma T_1^4(0)}, \tag{16}$$

which for an isothermal cavity (T_i = const) represent the local apparent emissivity of the cavity walls [1]. We further assume that the temperatures $T_1(r) = T_1$ = const, $T_3(r^*) = T_3$ = const, and that the temperature of surface 2 varies linearly.

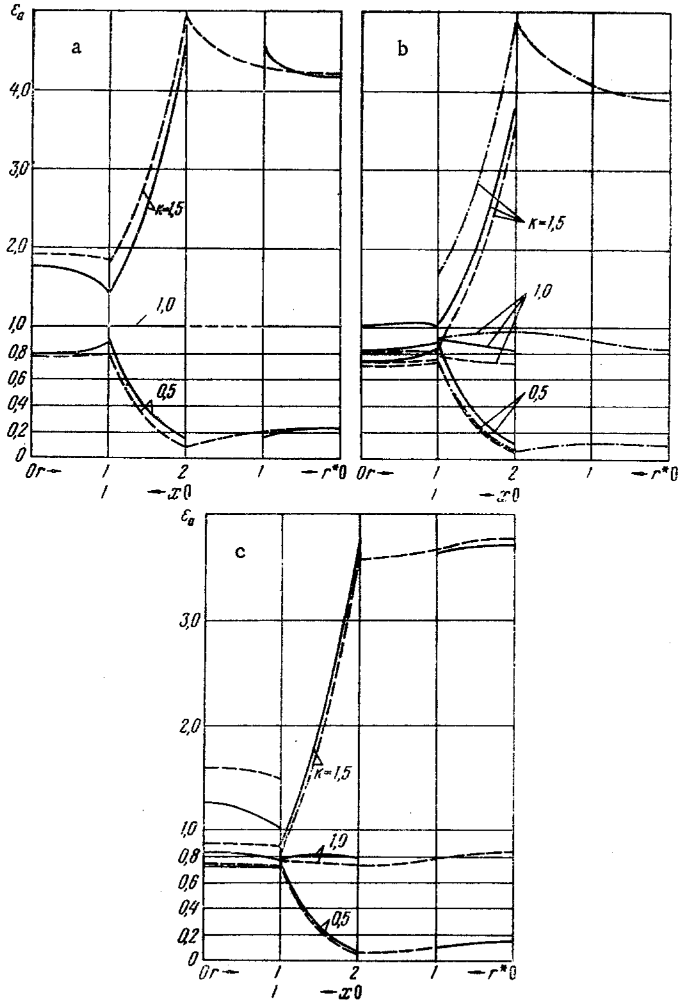

Fig. 2. Apparent emissivity (L = 1.0, ε = 0.7): a) closed cavity (α = 45°, dashed curve; α = 0°, solid curve); b) cavities open at one end (α = 45°, dashed and dot-dashed curves; α = 0°, solid curve); c) cavities open at both ends, and two disks (α = 45°, dashed curve; α = 0° solid curve).

Let $T_3 = kT_1$; then

$$T_2(x) = T_1[(1-k)x + k] = T_1 f(k,\ x). \tag{17}$$

In addition, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$ (except for the cases considered in Fig. 1b).

System (3)-(5) then becomes

$$\varepsilon_a(r_0) = \varepsilon + (1-\varepsilon)\left[\int_0^L \varepsilon_a(x)\,dF^*_{r_0-x} + \int_0^{R^*} \varepsilon_a(r^*)\,dF^*_{r_0-r^*}\right], \tag{3'}$$

$$\varepsilon_a(x_0) = \varepsilon f^4(k,\ x) + (1-\varepsilon)\left[\int_0^1 \varepsilon_a(r)\,dF^*_{x_0-r} + \int_0^L \varepsilon_a(x)\,dF^*_{x_0-x} + \right.$$

$$\left. + \int_0^{R^*} \varepsilon_a(r^*)\,dF^*_{x_0-r^*}\right], \tag{4'}$$

$$\varepsilon_a(r_0^*) = \varepsilon k^4 + (1-\varepsilon)\left[\int_0^1 \varepsilon_a(r)\,dF^*_{r_0-r} + \int_0^L \varepsilon_a(x)\,dF^*_{r_0^*-x}\right]. \tag{5'}$$

We solve (3')-(5') numerically by the iteration method. To do this we divide the segments [0, 1], [0, L], and [0, R*] into 50 equal parts. The distances between node points are $h_r = 0.02$, $h_x = 0.02$ L and $h_{r*} = 0.02R*$, respectively. Each integral occurring in (3')-(5') is replaced by the sum of 25 integrals that can be evaluated between 0 and $2h_r$, $2h_r$ and $4h_r$, etc. Equation (3') is then written as

$$\varepsilon_a(r_0) = \varepsilon + (1-\varepsilon)\sum_{n=1}^{25}\Big[\int\limits_{(2n-2)h_x}^{2nh_x} \varepsilon_a(x)\, dF^*_{r_0-x} + \int\limits_{(2n-2)h_{r^*}}^{2nh_{r^*}} \varepsilon_a(r^*)\, dF^*_{r_0-r^*}\Big]. \quad (3'')$$

Equations (4') and (5') are of similar form (we call them (4") and (5")). On each segment of integration ε_a the unknown functions [(2n − 2)h, 2nh] are approximated by a polynomial of degree two; the reference points for this approximation are the values of the functions ε_a at the node points (2n − 2)h, (2n − 1)h, and 2nh.

Each integral $\int\limits_{(2n-2)h}^{2nh} \varepsilon_a\, dF^*$ is evaluated by Simpson's rule with automatic choice of step.

The iterations are organized as follows: the functions $\varepsilon_a(x) = \varepsilon f^4(k,\ x)$, $\varepsilon_a(r^*) = \varepsilon k^4$ are used as the zeroth approximation and (3") is calculated, which yields $\varepsilon^1_a(r)$. We next use this function to calculate (5") (the function $\varepsilon^1_a(r^*)$). In the last step the calculated functions $\varepsilon^1_a(r)$ and $\varepsilon^1_a(r^*)$ are used to solve (4"). The computational procedure then repeats. The process terminates when the condition

$$\left|\frac{\varepsilon^j_a - \varepsilon^{j-1}_a}{\varepsilon^{j-1}_a}\right| < 0{,}005, \quad (A)$$

is satisfied, where j is the number of the iteration.

The singularities at the corner points x = 0, r* = R*, and x = L, r = 1 are bypassed in the following manner: for (3"), for example, on each iteration calculations are carried out at the nodes O, h_r, $2h_r$,... $(n-1)h_r$, and at the node point $nh_r = 1$ the value of the function ε_a (1) is found by extrapolation; a polynomial of second degree is used which is constructed from the reference points at the nodes $(n-3)h_r$, $(n-2)h_r$, and $(n-1)h_r$. An analogous procedure was used to solve (4") and (5"). It took 3-11 iterations to solve the problem.

Calculations were carried out for the range of cavities shown in Fig. 1b. We took k = 0.5; 1.0; 1.5 (see (17)), ε = 0.7, α = 0°, and 45°, L = 1.0.

Analyzing the results shown in Fig. 2(a, b, c), we conclude the following: for closed cavities (see Fig. 2a) with an isothermal wall (T_i = const) the result is trivial: the emissivity is $\varepsilon_a = 1$ for any point of the cavity. For k ≠ 1 the temperature distribution along the cavity axis begins to play an important role; the geometric factor (cavity flare angles of α = 0° and 45°) does not have much influence and leads to a maximum difference of 30%.

Figure 2b shows the calculated results for cavities open at one end. The solid and dashed lines represent data for a cavity with the open end to the right (α = 0° and 45°, respectively); the dash-dotted lines represent data for a cavity open to the left (α = 45°). The α = 0° case (isothermal cavity) served as the model problem and was compared with the results of [1]. The results are in full agreement.

Here, as before, the decisive role was played by the temperature distribution; a geometric factor such as the cavity flare angle had far less influence.

The results calculated for a cavity open at both ends (α = 0° and 45°) are plotted at the center of Fig. 2c; the sides of the figure show data for heat transfer between two disks.

The data of Fig. 2a, b, c are universal in nature and permit rapid estimation of radiation heat transfer (with the aid of (1), (16)).

NOTATION

T	is the temperature;
σ	is the Stefan–Boltzmann constant;
ε	is the emissivity;
r, r*, x, φ	are the running coordinates;
B	is the effective radiation flux;
H	is the incident radiation heat flux;
dF*	is the elementary angular coefficient;
β	is the angle determining the orientation of the normal to the elementary area;
s	is the distance between elementary areas;

dA	is the value of an elementary area;
R, L, R*, α	are parameters determining the shape of the cavity;
ε_a	is the apparent emissivity;
k	is the ratio between cavity end-surface temperatures;
f(k, x)	is the function determining the temperature distribution along the lateral surface of the cavity;
h	is the distance between node points in the numerical solution of the system of integral equations;
i	is the surface number;
j	is the iteration number.

LITERATURE CITED

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